Some Improved Intuitionistic Fuzzy Confidence Weighted Operators and Their Application in the Evaluation of the Teaching Quality

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Abstract—To incorporate the decision makers’ confidence levels in the decision making process, in this study, we shall propose some intuitionistic fuzzy confidence weighted geometric operators, including the improved intuitionistic fuzzy confidence weighted averaging (IIFCWA) operator, the improved intuitionistic fuzzy confidence weighted geometric (IIFCGW) operator, the improved generalized intuitionistic fuzzy confidence weighted averaging (IGIFCWA) operator and the improved generalized intuitionistic fuzzy confidence weighted geometric (IGIFCGW) operator. Some desirable properties and special cases of the proposed aggregation operators are also discussed. Finally, based on the new aggregation operators, a novel approach is proposed to solve the multiple attribute decision making with the intuitionistic fuzzy information, and a numerical example is given to illustrate feasibility of the proposed method.

Keywords—intuitionistic fuzzy set, decision making, confidence level, aggregating operator, teaching quality

I. INTRODUCTION

In some decision making problems, the decision makers may be can not evaluate the alternatives with crisp numbers because of time pressure or lack of data, and they often prefer to use the intuitionistic fuzzy sets [1] to express their preference values. Therefore, how to aggregate the discrete intuitionistic fuzzy sets into a collective one is very important in the multiple attribute decision making (MADM), and it has attracted many attentions from the researchers [2-10]. In the past few decades, various aggregation operators have been proposed, which can be roughly classified into two types. The first type is based on the arithmetic mean. For example, Xu [2] developed the intuitionistic fuzzy weighted averaging (IFWA) operator, based on which, many aggregation operators are proposed, such as the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator and the intuitionistic fuzzy hybrid averaging (IFHA) operator in [2]. Then, based on the generalized mean or more general quasi mean, some generalized means or quasi means of the above three aggregation operators are introduced in [3], respectively. The second type is based on the geometric mean. In fact, all of the above aggregation operators have a corresponding geometric operator. For example, in [4], Xu and Yager introduced the intuitionistic fuzzy weighted geometric (IFWG) operator and the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator. Also in [4], based on the IFWG and IFOWG operators, Xu and Yager developed the intuitionistic fuzzy hybrid geometric (IFHG) operator.

However, as pointed in [5], all of the above aggregation operators don’t consider the confidence levels of the aggregated arguments, which are provided by the decision makers. Then, Yu [5] defined a generalized intuitionistic fuzzy confidence weighted geometric operator, which incorporates the confidence levels \( l_j (j = 1,2,\cdots,n) \) in its expression, which replaces the weighting \( \omega_j (j = 1,2,\cdots,n) \) in the classical generalized intuitionistic fuzzy weighted geometric (GIFWG) operator by \( l_j \times \omega_j (j = 1,2,\cdots,n) \). This is reasonable, because the larger of the confidence level \( l_j \), the more confidence of the decision maker about his/her preference value, thus it should impose more impact on the aggregated result. In the following Section 2, a simple example, indicates that the aggregating result becomes larger hen the confidence levels \( l_j (j = 1,2,\cdots,n) \) is smaller than one, which is inconsistent with intuition. To deal with this issue, in this paper, we shall define some improved confidence weighted geometric operators, such as the improved intuitionistic fuzzy confidence weighted averaging (IFCWA) operator, the improved intuitionistic fuzzy confidence weighted geometric (IFCGW) operator, the improved generalized intuitionistic fuzzy confidence weighted averaging (IGIFCWA) operator and the improved generalized intuitionistic fuzzy confidence weighted geometric (IGIFCGW) operator. Some desirable properties and special cases of the proposed aggregation operators are also discussed.

The remainder of the paper is organized as follows. In Section 2, we briefly review some basic concepts related to the intuitionistic fuzzy sets and some classical aggregation operators. Section 3 defines some improved intuitionistic fuzzy confidence weighted aggregation operators, and discusses their properties and special cases. In Section 4, a novel approach is proposed to solve the multiple attribute decision making with the intuitionistic fuzzy information, and a numerical example is given to illustrate feasibility of the proposed method. In Section 5, we give a brief conclusion.
II. PRELIMINARIES

The notion of intuitionistic fuzzy set (IFS) proposed by Atanassov [1] in 1986 is a useful tool to express fuzziness and uncertainty, which is characterized by a membership function and a non-membership function as follows.

**Definition 1** [4]. Let \( X \) be a universe of discourse, then the concept of intuitionistic fuzzy set (IFS) \( A \) on \( X \) is defined as

\[
A = \{ x, \mu_A(x), \nu_A(x) \mid x \in X \}
\]  

where \( \mu_A(x) \) and \( \nu_A(x) \) are mappings from \( X \) to the closed interval \([0,1]\) such that \( 0 \leq \mu_A(x) \leq 1, 0 \leq \nu_A(x) \leq 1 \) and \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \), \( \forall x \in X \), and they are called the degrees of membership and non-membership of element \( x \in X \) to the set \( A \), respectively.

Xu [2] defined the tuple \((\mu_\alpha, \nu_\alpha)\) as the intuitionistic fuzzy value (IFV), and denoted by \( \alpha \). Let \( Q \) denote the set of all the IFVs. In addition, let \( h(\alpha_i) = \mu_{\alpha_i} + \nu_{\alpha_i} \) (\( i = 1, 2 \)) be the accuracy degrees of \( \alpha_i (i = 1, 2) \) and \( s(\alpha_i) = \mu_{\alpha_i} - \nu_{\alpha_i} \) (\( i = 1, 2 \)) be the scores of \( \alpha_i (i = 1, 2) \). Xu and Yager [7] gave a total order relation between two IFVs \( \alpha_1 \) and \( \alpha_2 \), as follows:

- If \( s(\alpha_1) = s(\alpha_2) \), then \( \alpha_1 = \alpha_2 \).
- If \( s(\alpha_1) < s(\alpha_2) \), then \( \alpha_1 \) is smaller than \( \alpha_2 \), denoted by \( \alpha_1 < \alpha_2 \).

Moreover, the operational laws of the IFVs are defined as follows.

**Definition 2** [2]. Let \( \alpha = (\mu_\alpha, \nu_\alpha) \) and \( \beta = (\mu_\beta, \nu_\beta) \) be two IFVs, then

\[
\alpha \bigoplus \beta = (\mu_\alpha + \mu_\beta - \mu_\alpha \cdot \mu_\beta, \nu_\alpha \cdot \nu_\beta) \quad (1)
\]

\[
\alpha \bigotimes \beta = (\mu_\alpha \cdot \mu_\beta + \nu_\alpha - \nu_\alpha \cdot \nu_\beta) \quad (2)
\]

\[
\lambda \alpha = (1 - (1 - \mu_\alpha)^\lambda, (1 - \nu_\alpha)^\lambda), \lambda > 0 \quad (3)
\]

\[
\alpha^\lambda = (\mu_\alpha^\lambda, 1 - (1 - \nu_\alpha)^\lambda), \lambda > 0 \quad (4)
\]

The intuitionistic fuzzy weighted averaging/geometric operator proposed by Xu [2] and Xu and Yager [4] are two basic aggregation operators, which are defined as follows.

**Definition 3** [2]. Let \( \alpha = (\omega_1, \omega_2, \ldots, \omega_n) \) be a collection of IFVs, and \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) be the weight vector of \( \alpha_j (j = 1, 2, \ldots, n) \), where \( \omega_j \) indicates the importance degree of \( \alpha_j \), satisfying \( \omega_j > 0 (j = 1, 2, \ldots, n) \) and \( \sum_{j=1}^{n} \omega_j = 1 \). If

\[
\text{IFWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigoplus_{j=1}^{n} \omega_j \alpha_j,
\]

then \( \text{IFWA} \) is called the intuitionistic fuzzy weighted averaging (IFWA) operator. If

\[
\text{IFWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigotimes_{j=1}^{n} \alpha_j^{\omega_j},
\]

then \( \text{IFWG} \) is called the intuitionistic fuzzy weighted geometric (IFWG) operator.

In some practical decision making problems, the decision makers are often asked to present the familiarity degrees or the confidence levels about the evaluated object. Therefore, in [5], Yu defined the following generalized intuitionistic fuzzy confidence weighted geometric (GIFCWG) operator, which can deal with this issue.
Definition 4 [5]. Let $\alpha_j = (\mu_{\alpha_j}, v_{\alpha_j})(j = 1, 2, \ldots, n)$ be a collection of IFVs, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^\top$ be the weight vector of $\alpha_j (j = 1, 2, \ldots, n)$, where $\omega_j$ indicates the importance degree of $\alpha_j$, satisfying $\omega_j > 0 (j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{n} \omega_j = 1$. Let $l_j (j = 1, 2, \ldots, n)$ be the confidence levels of IFV $\alpha_j$ with $0 \leq l_j \leq 1$. If

$$GIFCWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \frac{1}{\lambda} \left( \bigotimes_{j=1}^{n} (\lambda \alpha_j)^{l_j/\omega_j} \right) = \left( 1 - \prod_{j=1}^{n} \left( 1 - (1 - \mu_{\alpha_j})^{(l_j/\omega_j)} \right)^{\lambda/\omega_j} \right) \left( 1 - \prod_{j=1}^{n} \left( 1 - v_{\alpha_j}^{(l_j/\omega_j)} \right)^{\lambda/\omega_j} \right), \quad (4)$$

where $\lambda > 0$, then GIFCWG is called the generalized intuitionistic fuzzy confidence weighted geometric (GIFCWG) operator.

Thought the GIFCWG operator is more powerful to handle the uncertain problems, the following example indicates that the aggregating result of the GIFCWG operator may be not reasonable.

Example 1. Let $\lambda = 1, \alpha_j = \alpha = (0.8, 0.1)(j = 1, 2, 3)$, and $l_j = l = 0.8(j = 1, 2, 3), \omega = (0.2, 0.6, 0.2)^\top$. Then

$$GIFCWG(\alpha_1, \alpha_2, \alpha_3) = (0.8365, 0.0808) > (0.8, 0.1) = \alpha.$$

The confidence levels $l_j (j = 1, 2, 3)$ of the decision maker are all smaller than 1, which indicate that the decision maker is not very confident with the evaluated object. Thus, the aggregating result should be smaller than the input arguments $\alpha_i (i = 1, 2, 3)$. However, the aggregating result is $(0.8365, 0.0808)$ is larger than $(0.8, 0.1)$.

From Eq.(4), we find that the degree of membership is a monotone increasing function with respect to the confidence level $l_j$, and the degree of nonmembership is a monotone decreasing function with respect to the confidence level $l_j$. Thus, the aggregating result becomes larger as the confidence level $l_j$ decreases. Furthermore, Example 1 also indicates that the GIFCWG property does not satisfy the idempotency property, i.e., $GIFCWG(\alpha, \alpha, \alpha) \neq l \alpha$.

III. CONFIDENCE AGGREGATION OPERATORS

In this section, in view of the problem exists in the GIFCWG operator, we shall introduce some improved confidence aggregation operators, including the IIFCWA, IIFCWG, IGIFCWA and IGIFCWG operators, which consider the decision makers' confidence levels with the object to be evaluated, and if the confidence levels $l_j = l (j = 1, 2, \ldots, n)$, then the above improved confidence aggregation operators reduces to the corresponding classical IFWA, IFWG, GIFWA and GIFWG operators, respectively. Now, we firstly give the definition of the improved intuitionistic fuzzy confidence weighted geometric (IIFCWG) operator.

Definition 5. Let $\alpha_j = (\mu_{\alpha_j}, v_{\alpha_j})(j = 1, 2, \ldots, n), \omega = (\omega_1, \omega_2, \ldots, \omega_n)^\top$ and $l_j (j = 1, 2, \ldots, n)$ be the same input in Definition 4. If

$$IIFCWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigotimes_{j=1}^{n} \left( \omega_j l_j \alpha_j \right) = \left( 1 - \prod_{j=1}^{n} \left( 1 - (1 - \mu_{\alpha_j})^{(l_j/\omega_j)} \right)^{\lambda/\omega_j} \right) \left( 1 - \prod_{j=1}^{n} \left( 1 - v_{\alpha_j}^{(l_j/\omega_j)} \right)^{\lambda/\omega_j} \right), \quad (5)$$

then \text{IIFCWA} is called the improved intuitionistic fuzzy confidence weighted averaging (IIFCWA) operator.

Example 2. Considering the IFVs in Example 1, we have

$$IIFCWA(\alpha_1, \alpha_2, \alpha_3) = (0.7241, 0.1585) = l \alpha < \alpha = (0.8, 0.1).$$

This is reasonable, because the confidence levels $l_j (j = 1, 2, 3)$ is smaller than 1, thus the aggregating result should be smaller than the input arguments.

Remark 1. If all the confidence levels $l_j (j = 1, 2, \ldots, n)$ equal to 1, then the IIFCWA operator reduces to the
intuitionistic fuzzy weighted averaging (IFWA) operator in [2].

**Remark 2.** From Eq. (5), it is obvious that the degree of membership of the aggregating result is a monotone decreasing function with respect to the confidence level, and the degree of nonmembership of the aggregating result is a monotone increasing function with respect to the confidence level.

The IIFCWG operator has the following desirable property.

**Property 1.** (Idempotency) If all \( \alpha_j (j = 1, 2, \ldots, n) \) are equal, i.e., \( \alpha_j = \alpha \) for all \( j \), and the confidence levels \( l_j = l(j = 1, 2, \ldots, n) \), then

\[
\text{IIFCWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = l\alpha.
\]

Based on the geometric mean and the IIFCWA operator, we now give the definition of the improved intuitionistic fuzzy confidence weighted geometric (IIFCWG) operator.

**Definition 6.** Let \( \alpha_j = (\mu_{\alpha_j}, v_{\alpha_j}) (j = 1, 2, \ldots, n) \), \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^\top \) and \( l_j (j = 1, 2, \ldots, n) \) be the same input in Definition 4. If

\[
\text{IIFCWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \prod_{j=1}^{n} \left( l_j \alpha_j \right)^{\omega_j} = \left( \prod_{j=1}^{n} \left( 1 - (1 - \mu_{\alpha_j})^{\omega_j} \right) \right)^{1/\lambda}, \quad \left( 1 - \prod_{j=1}^{n} (1 - v_{\alpha_j})^{\omega_j} \right)^{1/\lambda),}
\]

then **IIFCWG** is called the improved intuitionistic fuzzy confidence weighted geometric (IIFCWG) operator.

**Example 3.** Considering the IFVs in Example 1, we have

\[
\text{IIFCWG}(\alpha_1, \alpha_2, \alpha_3) = (0.7241, 0.1585) < (0.8, 0.1) = \alpha.
\]

**Remark 3.** If all the confidence levels \( l_j (j = 1, 2, \ldots, n) \) equal to 1, then the IIFCWG operator reduces to the intuitionistic fuzzy weighted geometric (IFWG) operator in [7].

The IIFCWG operator also has the following desirable property.

**Property 2.** (Idempotency) If all \( \alpha_j (j = 1, 2, \ldots, n) \) are equal, i.e., \( \alpha_j = \alpha \) for all \( j \), and the confidence levels \( l_j = l(j = 1, 2, \ldots, n) \), then

Now, we generalize the IIFCWA and IIFCWG operators by the generalized mean, and get the following two aggregation operators.

**Definition 7.** Let \( \alpha_j = (\mu_{\alpha_j}, v_{\alpha_j}) (j = 1, 2, \ldots, n) \), \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^\top \) and \( l_j (j = 1, 2, \ldots, n) \) be the same input in Definition 4. If

\[
\text{IGIFCWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left( \prod_{j=1}^{n} \left( 1 - (1 - \mu_{\alpha_j})^{\omega_j} \right) \right)^{1/\lambda}, \quad \left( 1 - \prod_{j=1}^{n} (1 - v_{\alpha_j})^{\omega_j} \right)^{1/\lambda),}
\]

where \( \lambda > 0 \), then **IGIFCWA** is called the improved generalized intuitionistic fuzzy confidence weighted averaging (IGIFCWA) operator.

To better understand the IGIFCWA operator, we give an example as follows.

**Example 4.** Let \( \lambda = 1, \alpha_1 = (0.7, 0.1), \alpha_2 = (0.8, 0.2), \alpha_3 = (0.5, 0.1), \alpha_4 = (0.6, 0.2) \), and the confidence levels is \((0.7, 0.9, 0.8, 0.9)\), and the weighting vector is \( \omega = (0.2, 0.3, 0.2, 0.3)^\top \). Then

\[
\text{IGIFCWA}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (0.6176, 0.2102).
\]

For Example 4, if the parameter \( \lambda \) of the IGIFCWA operator takes different values, we get different aggregated results and different scores. Fig. 1 shows the scores of the aggregated results is monotone increasing with respect to the parameter \( \lambda \).
In the following, some special cases of the IGIFCWA operator can be obtained as the change of the parameter $\lambda$.

1. If $\lambda = 1$, then we have $\text{IGIFCWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigoplus_{j=1}^{n} \omega_j \alpha_j$, which is the IIFCWA operator defined by Eq.(4).

2. If $\lambda = 2$, then we have $\text{IGIFCWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left( \bigoplus_{j=1}^{n} \omega_j \alpha_j \right)^2$, which is called the intuitionistic fuzzy confidence square mean operator.

**Remark 4.** If $I_j = 1(j = 1, 2, \ldots, n)$, then the IGIFCWA operator reduces to the GIFWA operator in [5]. That is

$$\text{GIFWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left( \bigoplus_{j=1}^{n} \omega_j \alpha_j \right)^{1/2}.$$ 

Similarly, based on the geometric mean and the IGIFWG operator, we can develop the following generalized type operator.

**Definition 8.** Let $\alpha_j = (\mu_{a_j}, v_{a_j})(j = 1, 2, \ldots, n)$, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^\top$ and $I_j(j = 1, 2, \ldots, n)$ be the same input in Definition 4. If

$$\text{IGIFCWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \frac{1}{\lambda} \left( \bigotimes_{j=1}^{n} (\lambda I_j \alpha_j)^{\omega_j} \right)^{1/2} \left( 1 - \prod_{j=1}^{n}(1 - (1 - \mu_{a_j})^{\omega_j})^{1/2} \right)^{1/2}, $$

where $\lambda > 0$, then $\text{IGIFCWG}$ is called the improved generalized intuitionistic fuzzy confidence weighted geometric (IGIFCWG) operator.

**Example 5.** Considering the IFVs in Example 4, we have $\text{IGIFCWG}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (0.1411, 0.6984)$. 

Figure 1. Scores of aggregated results obtained by the IGIFCWA operator

Figure 2. Scores of aggregated results obtained by the IGIFCWG operator
For Example 5, if the parameter $\lambda$ of the IGIFCWG operator takes different values, we also get different aggregated results and different scores. Fig. 2 shows the scores of the aggregated results is monotone decreasing with respect to the parameter $\lambda$.

In the following, some special cases of the IGIFCWG operator can be obtained as the change of the parameter $\lambda$.

(1). If $\lambda = 1$, then we have
$$\text{IGIFCWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigotimes_{j=1}^{n} \left( l_j \alpha_j \right)^{\alpha_j},$$
which is the IIFCWG operator defined by Eq.(5).

(2). If $\lambda = 2$, then we have
$$\text{IGIFCWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \frac{1}{2} \bigg( \bigotimes_{j=1}^{n} (2 l_j \alpha_j)^{\alpha_j} \bigg),$$
which is called the intuitionistic fuzzy confidence geometric square mean operator.

Remark 5. If $l_j = 1 (j = 1, 2, \ldots, n)$, then the IGIFCWG operator reduces to the GIFWG operator in [5]. That is
$$\text{GIFWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \frac{1}{\lambda} \bigg( \bigotimes_{j=1}^{n} (\lambda \alpha_j)^{\alpha_j} \bigg).$$

IV. THE APPLICATION OF THE IGIFCWG OPERATOR

For a multiple attribute decision making problems with intuitionistic fuzzy information, let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of $m$ alternatives, and $G = \{G_1, G_2, \ldots, G_n\}$ be the set of $n$ attributes, whose weigh vector is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^{\top}$, $l_j \in (0,1](j = 1, 2, \ldots, n)$ is the confidence level, which indicates that the degree of confidence or familiarity of the decision maker with respect to the attribute $G_j (j = 1, 2, \ldots, n)$. Suppose that $\tilde{R} = (\alpha_{ij})_{m \times n}$ is the decision matrix, where $\alpha_{m\times n}$ is a preference value, which takes the form of the intuitionistic fuzzy variable, given by the decision maker for the alternative $A_i$ with respect to the attribute $G_j \in G$. Then, based on the IGIFCWG operator or other three new aggregation operators, we will propose an approach to solve this multiple attribute decision making problem, which involves the following procedures.

Step 1. Utilize the IGIFCWG operator:
$$\tilde{r}_i = \text{IGIFCWG}(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}) = \frac{1}{\lambda} \bigg( \bigotimes_{j=1}^{n} (\lambda l_j \alpha_j)^{\alpha_j} \bigg), i = 1, 2, \ldots, m,$$

to aggregate all the preference values, $\alpha_{ij} (j = 1, 2, \ldots, n)$, of the $i$th row, and get the overall preference value $\tilde{r}_i$, which is correspondent to the alternative $A_i$.

Step 2. Calculate the scores of $\tilde{r}_i (i = 1, 2, \ldots, m)$, and utilize the comparison method of IFVs described in Section 2 to rank these collective preference values $\tilde{r}_i (i = 1, 2, \ldots, m)$.

Step 3. Rank all the alternatives $A_i (i = 1, 2, \ldots, m)$ in accordance with the collective overall preference values $\tilde{r}_i (i = 1, 2, \ldots, m)$ and select the best one(s).

Step 4. End.

Example 6. The teaching quality is of significant importance to every university, which is related to their sustainable development. The teaching work can be well guided by universities internal self-evaluation. Now, as a basic required course for almost all students in every universities, Advanced Mathematics is in charge of the School of Mathematics and Statistics and it is mainly facing to the freshmen. Suppose that there are five lectures teach the course and they are denoted by $A_i (i = 1, 2, \ldots, 5)$. They are evaluated from four aspects, including teaching effect $G_1$; teaching method $G_2$; teaching designing $G_3$ and teaching content $G_4$. The weighting vector of the four attributes is $\omega = (0.2, 0.3, 0.3, 0.2)^{\top}$

And the decision matrix is present in the form of intuitionistic fuzzy values listed in Table I.
TABLE I. INTUITIONISTIC FUZZY DECISION INFORMATION

<table>
<thead>
<tr>
<th>$A_i$</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.9,0.0)</td>
<td>(0.5,0.4)</td>
<td>(0.8,0.1)</td>
<td>(0.5,0.4)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.7,0.2)</td>
<td>(0.9,0.0)</td>
<td>(0.6,0.3)</td>
<td>(0.8,0.1)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.4,0.5)</td>
<td>(0.7,0.2)</td>
<td>(0.9,0.0)</td>
<td>(0.7,0.2)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.6,0.3)</td>
<td>(0.6,0.3)</td>
<td>(0.7,0.2)</td>
<td>(0.8,0.1)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.9,0.1)</td>
<td>(0.7,0.1)</td>
<td>(0.6,0.2)</td>
<td>(0.9,0.1)</td>
</tr>
</tbody>
</table>

The confidence levels vector is $(0.9, 0.8, 0.8, 0.9)^T$. Table II shows the different aggregated results of the five alternatives by the IGIFCWG operator as the parameter $\lambda$ changes.

TABLE II. AGGREGATED RESULTS

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 1$</td>
<td>(0.5865,0.3048)</td>
<td>(0.6809,0.2012)</td>
<td>(0.6199,0.2621)</td>
<td>(0.6007,0.2957)</td>
<td>(0.6740,0.1833)</td>
</tr>
<tr>
<td>$\lambda = 2$</td>
<td>(0.5604,0.3479)</td>
<td>(0.6610,0.2448)</td>
<td>(0.5932,0.3126)</td>
<td>(0.5941,0.3063)</td>
<td>(0.6479,0.1926)</td>
</tr>
<tr>
<td>$\lambda = 3$</td>
<td>(0.5389,0.3747)</td>
<td>(0.6420,0.2713)</td>
<td>(0.5676,0.3463)</td>
<td>(0.5877,0.3155)</td>
<td>(0.6271,0.2027)</td>
</tr>
<tr>
<td>$\lambda = 4$</td>
<td>(0.5221,0.3930)</td>
<td>(0.6255,0.2900)</td>
<td>(0.5448,0.3731)</td>
<td>(0.5820,0.3225)</td>
<td>(0.6118,0.2123)</td>
</tr>
<tr>
<td>$\lambda = 5$</td>
<td>(0.5092,0.4059)</td>
<td>(0.6119,0.3038)</td>
<td>(0.5248,0.3950)</td>
<td>(0.5770,0.3282)</td>
<td>(0.6005,0.2206)</td>
</tr>
</tbody>
</table>

Table III shows the scores and rankings of the five alternatives as the parameter $\lambda$ changes.

TABLE III. SCORES AND RANKINGS

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
<th>Rankings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 1$</td>
<td>0.2871</td>
<td>0.4796</td>
<td>0.3578</td>
<td>0.3050</td>
<td>0.4908</td>
<td>$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$</td>
</tr>
<tr>
<td>$\lambda = 2$</td>
<td>0.2124</td>
<td>0.4163</td>
<td>0.2806</td>
<td>0.2878</td>
<td>0.4553</td>
<td>$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$</td>
</tr>
<tr>
<td>$\lambda = 3$</td>
<td>0.1641</td>
<td>0.3707</td>
<td>0.2214</td>
<td>0.2725</td>
<td>0.4244</td>
<td>$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$</td>
</tr>
<tr>
<td>$\lambda = 4$</td>
<td>0.1291</td>
<td>0.3355</td>
<td>0.1717</td>
<td>0.2596</td>
<td>0.3995</td>
<td>$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$</td>
</tr>
<tr>
<td>$\lambda = 5$</td>
<td>0.1033</td>
<td>0.3082</td>
<td>0.1298</td>
<td>0.2488</td>
<td>0.3798</td>
<td>$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$</td>
</tr>
</tbody>
</table>

From the numerical results in Table III, we find that: (1) the score of every alternatives is decreasing with respective to the parameter $\lambda$. This feature is crucial in real decision making, and we can take the parameter as small as possible if the decision maker is risk preference; (2) the different ranking results are obtained by assigning different values of the parameter $\lambda$; (3) the best alternative is consistently the same, and the alternative $A_5$ is always the best alternative for all the given $\lambda$.

REFERENCES
