The Application of Cubic Spline Interpolation Method in Gas Logging Interpretation

Wu Xiaoyu, Zhang Fangzhou, She Tianwei
Changshu Institute of Technology, Jiangsu Province, 215500, China

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Abstract: In the logging technology, gas logging is an important method for reservoir evaluation, gas logging technology includes Kessler hydrocarbon ratio method and triangle plate method. However, according to the theory of experience to select the value point, value point error will be generated. Therefore, cubic spline interpolation method is proposed to select the value area of interpretation plate, the method is applied to reservoir evaluation. Cubic spline interpolation method is studied to calculate the value point, the theory and method of gas logging technology are studied. The value area obtained by the cubic spline interpolation method is used in triangle chart of gas logging technology, this can improve the accuracy of recognition of oil and gas reservoirs.

1. Introduction
At present, the hydrocarbon data detected by the gas logging technology is interfered by many factors, and the value points exist errors which obtain in triangle plate. Therefore, we should use cubic spline interpolation method, the thought of the parameter interpolation, and choosing the value area of the gas logging’s key drawing to reduce the error of the gas logging interpretation and improving the accuracy of reservoir evaluation.

2. Cubic spline interpolation method
The cubic spline interpolation method’s mathematical description is as follows:
Suppose \( y = f(x) \) has a set of nodes \( a = x_0 < x_1 < x_2 < \ldots < x_n = b \) on the interval \([a, b]\), and the corresponding function values are \( y_0, y_1, \ldots, y_n \), if \( S(x) \) has the following properties:

1) each sub-interval \([x_{i-1}, x_i]\), \( i = 1, 2, 3, \ldots, n \) the \( S(x) \) is not higher than the cubic polynomial;

2) where \( S(x), S'(x), S''(x) \) is continuous on interval \([a, b]\), and \( S(x) \) is a cubic spline function.

3) \( S(x) \approx y_i \), \( i = 1, 2, 3, \ldots, n \), \( S(x) \) is \( y \)’s cubic spline function \([4]\).

2.1 The boundary conditions of cubic spline
The periodicity conditions of cubic spline interpolation method is\([5]\):
\[
M_1 = M_n, c_n M_2 + a_n M_{n-1} + 2M_n = d_n \quad (1)
\]
\[
a_n = \frac{h_n}{h_2 + h_n}, c_n = 1 - a_n \quad (2)
\]
\[
d_n = \frac{6}{h_2 + h_n} \left( \frac{\Delta y_2}{h_2} - \frac{\Delta y_n}{h_n} \right) \quad (3)
\]

Therefore, \( M_i(i = 1, 2, 3, \ldots, n) \)’s \( n \) equations are obtained. Introduce a \( n \)-dimensional vector which is called \( e \) and a \( n + 1 \) dimensional vector which is called \( f \).

2.2 Algorithm design of closed curve area
The following formulas can be gotten according to \( S(x), S'(x), S''(x) \) continuity at nod and
natural boundary condition:\[ s(x_i - 0) = s(x_i + 0) \]
\[ s'(x_i - 0) = s'(x_i + 0) \]
\[ s''(x_i - 0) = s''(x_i + 0) \]
\[ s''(x_0) = f''_0 = f''(x_0), s''(x_n) = f''_n = f''(x_n) \] (4)

The following formulas can be gotten according to \( S(x) \) is cubic polynomial on interval \([a, b]\) \( S'(x) \) is linear polynomial on interval \([a, b]\) and \( S''(x) = M_k \)
\[ s'(x) = \frac{x_{i+1}-x_i}{h_i} M_i + \frac{x-x_i}{h_i} M_i = 1 \] (6)

According to \( S(xi) = yi, S(xi+1) = yi + 1 \) and combining with the equation above:
\[ s(x) = \frac{(x_{i+1}-x)^3}{6h_i} M_i + \frac{(x-x_i)}{6h_i} M_i + Z \]
\[ Z = \left[ y_i - \frac{M_i h_i}{6} \right] \frac{x_{i+1}-x_i}{h_i} + \left[ y_{i+1} - \frac{M_{i+1} h_i}{6} \right] \frac{x-x_i}{h_i} \] (7)

The following formulas can be gotten according to is a continuous function \( S'(x) \) is a continuous function:\[ s'(x) = f[x_0, x_1] - f[x_{i-1}, x_i] \]
\[ s'(x_n) = f[x_{n-1}, x_n] - \frac{M_{n-1} + 2M_n}{6h_{n-1}} h_{n-1}^2 = f'_n \] (8)

The following formulas can be gotten according to equations above
\[ 2M_0 + M_1 = \frac{6}{h_0} (f[x_0, x_1] - f'_0) \]
\[ M_{n-1} - 1 + 2M_n = \frac{6}{h_{n-1}} (f'_n - f[x_{n-1}, x_n]) \]
\[ M_0 = f'_0, M_n = f'_n \] (9)

The cubic spline interpolation function can be obtained by solving the equations. In the general coordinate system, the drawing of closed curve region can be realized by using cubic spline interpolation method. As shown in figure 1

![Figure 1 Drawing of closed curve area](image_url)

3. The gas logging technique

3.1 Triangle plate

The gas logging interpretation triangle plate is a gas interpretation method based on hydrocarbon
components. It consists of equilateral triangle which consists of polar coordinates made up the ratio of $\frac{C2}{\sum C}, \frac{C3}{\sum C}, \frac{C4}{\sum C}$ and 60° polar angle, and inner angle triangle which consists of three parallel lines made up the data-point of $\frac{C2}{\sum C}, \frac{C3}{\sum C}, \frac{C4}{\sum C}$, as shown in figure 2.

![Figure 2 The gas logging interpretation triangle plate](image)

### 3.2 Drawing value area of triangle plate by cubic spline interpolation

Most interpolation methods assume that the interpolated curve is mapping relation, which is misunderstood to be each given $x$ has a corresponding $y$. In fact, some curves do not satisfy the one-to-one requirement. The closed curve area (the value area) is drawn in triangle plate, and the interpolation method is designed as follows:

1) Construct data Table $a = \{0.05, 0.5, 0.5, 0.5, \ldots, 0.5\}, b = \{0.05, 0.5, 0.5, 0.5, \ldots, 0.5\}$, according to equation (9), $\frac{dx}{\partial x} = 0, \frac{dx}{\partial x} = 3(x_{i+1} + x_{i-1}), \frac{dy}{\partial y} = 0, \frac{dy}{\partial y} = 3(y + 1 + y - 1)$ can be got.

2) Calculate $x$’s $M_x$ and $y$’s $M_y$ independently.

3) For any given parameter $t \in [a, b]$, using interpolation formula to calculate $S(x)$, this is the difference point of the closed curve. As shown in figure 3, drawing closed curves in triangle plate by interpolation method, that is, the value area in triangle plate.

![Figure 3 the drawing of value area in triangle plate](image)

### 3.3 Application example

The gas logging data with X depth of 3272 ~ 3283 are selected, and the gas logging data are sorted out by using the cubic spline interpolation method and the triangle plate method. (which
\[ \sum C = C_1 + C_2 + C_3 + iC_4 + nC_4 \], The criteria are as shown in Table 1.

<table>
<thead>
<tr>
<th>Large and inverted triangle</th>
<th>&gt;75</th>
<th>25~50</th>
<th>low yield reservoir</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suitable and inverted triangle</td>
<td>25-75</td>
<td>50~70</td>
<td>Oil reservoir</td>
</tr>
<tr>
<td>Small and inverted triangle</td>
<td>&lt;25</td>
<td>70-80</td>
<td>Hydrocarbon reservoir</td>
</tr>
<tr>
<td>Large and positive triangle</td>
<td>75~100</td>
<td>60~90</td>
<td>Oil-bear oil-bearing water reservoir</td>
</tr>
<tr>
<td>Large and positive triangle</td>
<td>&gt;100</td>
<td>90-100</td>
<td>Gas reservoir</td>
</tr>
</tbody>
</table>

Sing triangle chart and gas survey data to distinguish hydrocarbon reservoir, the value points can be screened intuitively according to the value area which are drawn. The results of gas logging data and interpretation are shown in Table 2.

<table>
<thead>
<tr>
<th>number</th>
<th>depth</th>
<th>(C_1)</th>
<th>(C_2)</th>
<th>(C_3)</th>
<th>(iC_4)</th>
<th>(nC_4)</th>
<th>conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3273</td>
<td>0.1053</td>
<td>0.0089</td>
<td>0.0058</td>
<td>0</td>
<td>0</td>
<td>non-productive</td>
</tr>
<tr>
<td>2</td>
<td>3274</td>
<td>0.0992</td>
<td>0.0039</td>
<td>0.0058</td>
<td>0</td>
<td>0</td>
<td>non-productive</td>
</tr>
<tr>
<td>3</td>
<td>3275</td>
<td>0.1239</td>
<td>0.0089</td>
<td>0.0058</td>
<td>0</td>
<td>0</td>
<td>non-productive</td>
</tr>
<tr>
<td>4</td>
<td>3276</td>
<td>0.5349</td>
<td>0.0681</td>
<td>0.0266</td>
<td>0.0014</td>
<td>0.0034</td>
<td>gas reservoir</td>
</tr>
<tr>
<td>5</td>
<td>3277</td>
<td>0.7359</td>
<td>0.0907</td>
<td>0.0497</td>
<td>0.0014</td>
<td>0.0091</td>
<td>gas reservoir</td>
</tr>
<tr>
<td>6</td>
<td>3278</td>
<td>4.2363</td>
<td>0.6464</td>
<td>0.2909</td>
<td>0.0181</td>
<td>0.0553</td>
<td>gas reservoir</td>
</tr>
<tr>
<td>7</td>
<td>3279</td>
<td>2.6127</td>
<td>0.4451</td>
<td>0.2445</td>
<td>0.0286</td>
<td>0.0463</td>
<td>hydrocarbon reservoir</td>
</tr>
<tr>
<td>8</td>
<td>3280</td>
<td>2.1061</td>
<td>0.3946</td>
<td>0.1581</td>
<td>0.0181</td>
<td>0.0371</td>
<td>hydrocarbon reservoir</td>
</tr>
<tr>
<td>9</td>
<td>3281</td>
<td>0.6548</td>
<td>0.1216</td>
<td>0.0983</td>
<td>0.0084</td>
<td>0.0276</td>
<td>oil reservoir</td>
</tr>
<tr>
<td>10</td>
<td>3282</td>
<td>0.7802</td>
<td>0.1341</td>
<td>0.983</td>
<td>0.0084</td>
<td>0.0276</td>
<td>hydrocarbon reservoir</td>
</tr>
</tbody>
</table>

It can be seen from the effect figure that the specific ratio point of the production reservoir falls inside the drawing closed area, and the ratio point of the non-production reservoir falls outside the value area, as shown in figure 4.

\[ \frac{C_1}{\Sigma C} = 13.403 \]
\[ \frac{C_2}{\Sigma C} = 9.384 \]
\[ \frac{nC_4}{\Sigma C} = 2.635 \]

Side length ratio = 27.75%
C. relative content = 73.78%

Figure 4 Impression drawing of triangle plate
4. Conclusion

The cubic spline interpolation method is used to draw the closed area of the triangle plate which belongs to the gas logging interpretation in this article. The gas logging interpretation chart is designed which use the theory of triangle plate and the interpretation standard, combing with the value area of the drawing. The results show that if the ratio point falls in the value area and it can meet the interpretation standard, the reservoir fluid property is produced layer. This method has practical application value.

References


78-84. (In Chinese)